

Two-flavor color superconductivity at finite temperature, chemical potential and in the presence of strong magnetic fields

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Utilizing an extended two-flavor Nambu-Jona Lasinio (NJL) model, we review some of the effects of external magnetic fields on two-flavor color superconducting phase (2SC) at moderate baryon densities in the QCD phase diagram. The effective action of the extended NJL model consists of two mass gaps as functions of three intensive quantities, the temperature, the quark chemical potential and the external magnetic field. The nonzero values of the mass gaps, chiral and diquark condensates, induce spontaneous chiral and color symmetry breaking, respectively, and as a result two different phases of quark matter appear. We find the transition curves between these phases as well as the critical points in the QCD phase diagram in terms of the intensive quantities. Imposing a constant strong magnetic field on these two phases, we show that the mass gaps increase with the magnetic field and the symmetry breaking region in the QCD phase diagram expands even to the larger values of temperature and quark chemical potential. This phenomenon is a consequence of the magnetic catalysis of dynamical symmetry breaking, which is proven before.

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1. Magnetized Two-Flavor NJL Model at Finite T and μ

The extended fermionic Lagrangian density of a two-flavor gauged NJL model is [1],

$$\mathcal{L}_f = \bar{\psi}(x)[i\gamma^\mu(\partial_\mu - i\tilde{e}\tilde{Q}\tilde{A}_\mu) + \mu\gamma^0]\psi(x) + G_S[(\bar{\psi}(x)\psi(x))^2 + (\bar{\psi}(x)i\gamma_5\vec{\tau}\psi(x))^2] + G_D[(i\bar{\psi}^C(x)\epsilon_f\epsilon_c^3\gamma_5\psi(x))(i\bar{\psi}(x)\epsilon_f\epsilon_c^3\gamma_5\psi^C(x))], \quad (1.1)$$

where, $\psi^C = C\bar{\psi}^T$ with $C = i\gamma^2\gamma^0$, $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are Pauli matrices, and $(\epsilon_c^3)^{ab} \equiv (\epsilon_c)^{ab3}$ and $(\epsilon_f)_{ij}$ are antisymmetric matrices in color and flavor spaces, respectively. Moreover, G_S (G_D) is the scalar (diquark) coupling. The rotated electric charge operator in the color-flavor space is, $\tilde{Q} = Q_f \otimes 1_c - 1_f \otimes \left(\frac{\lambda_8}{2\sqrt{3}}\right)_c$ and the rotated photon field is $\tilde{A}_\mu = A_\mu \cos \theta - G_\mu^8 \sin \theta$. Here, Q_f and λ_8 are the electric charge and the 8th Gell-Mann matrices respectively, and, A_μ (G_μ^8) is the photon (8th gluon) gauge field. Each quark degree of freedom has a rotated electric charge in units of $\tilde{e} = e \cos \theta$ with $\cos \theta = -\sqrt{3}g/\sqrt{3g^2 + e^2}$, where e and g are the electromagnetic and strong couplings, respectively. The rotated charges are $(\tilde{q}_{u_r} = 1/2, \tilde{q}_{u_g} = 1/2, \tilde{q}_{u_b} = 1, \tilde{q}_{d_r} = -1/2, \tilde{q}_{d_g} = -1/2, \tilde{q}_{d_b} = 0)$. Introducing the auxiliary fields, $\sigma = -2G_S(\bar{\psi}\psi)$, as meson and $\Delta = -2G_D(i\bar{\psi}^C\epsilon_f\epsilon_c^3\gamma_5\psi)$, as diquark fields to the partition function corresponding to (1.1) and eventually integrating out the fermions, we obtain the one-loop effective potential in terms of the mass gaps, σ and Δ , (see [1] for more details). Assuming that the external magnetic field is aligned in the third direction, $\vec{B} = B\mathbf{e}_3$, the magnetized effective potential at finite T and quark chemical potential μ is given by

$$\begin{aligned} \Omega_{\text{eff}} = & \frac{\sigma^2}{4G_S} + \frac{|\Delta|^2}{4G_D} + \frac{B^2}{2} - \frac{2}{\beta} \int \frac{d^3p}{(2\pi)^3} \left\{ \beta E_0 + \ln \left(1 + e^{-\beta(E_0+\mu)} \right) + \ln \left(1 + e^{-\beta(E_0-\mu)} \right) \right\} \\ & - \frac{2\tilde{e}B}{\beta} \sum_{n=0}^{+\infty} \alpha_n \int_{-\infty}^{+\infty} \frac{dp_3}{8\pi^2} \left\{ \beta E_{+1} + \ln \left(1 + e^{-\beta(E_{+1}+\mu)} \right) + \ln \left(1 + e^{-\beta(E_{+1}-\mu)} \right) \right\} \\ & - \frac{4\tilde{e}B}{\beta} \sum_{n=0}^{+\infty} \alpha_n \int_{-\infty}^{+\infty} \frac{dp_3}{16\pi^2} \left\{ \beta \left(|E_{\frac{+1}{2}}^+| + |E_{\frac{+1}{2}}^-| \right) + 2\ln \left(1 + e^{-\beta E_{\frac{+1}{2}}^+} \right) + 2\ln \left(1 + e^{-\beta E_{\frac{+1}{2}}^-} \right) \right\}. \quad (1.2) \end{aligned}$$

Here, $E_{\tilde{q}} = \sqrt{2|\tilde{q}\tilde{e}B|n + p_3^2 + \sigma^2}$, for $\tilde{q} = 1, \pm\frac{1}{2}$, $E_{\tilde{q}=0} = \sqrt{\vec{p}^2 + \sigma^2}$, $E_{\tilde{q}}^\pm = \sqrt{(E_{\tilde{q}} \pm \tilde{\mu})^2 + |\Delta|^2}$ and $\beta \equiv T^{-1}$ are considered. We use the Ritus method [2], to determine the fermion propagators, and introduce the Landau levels, n , with degeneracies, $\alpha_n = 2 - \delta_{n0}$. The vacuum expectation values of bosons are determined through global minimization of the effective potential. For numerical calculations, we use the parameters $\Lambda = 653.3$ MeV, $G_S = 5.0163$ GeV $^{-2}$, and $G_D = \frac{3}{4}G_S$, [3].

2. The Phase Diagram of Two-Flavor Quark Matter

The color-superconducting phase (CSC), where color symmetry is spontaneously broken and the chiral symmetry broken phase (χ SB) are characterized for nonzero $\tilde{e}B$ by $\sigma_B = 0, \Delta_B \neq 0$, and $\sigma_B \neq 0, \Delta_B = 0$, respectively. Defining these two phases and the normal phase by $\sigma_B = 0, \Delta_B = 0$, the two-flavor model phase diagrams in three spaces, $T_c - \mu$, $\mu_c - \tilde{e}B$ and $T_c - \tilde{e}B$ are shown in figures 1(a), (b) - 2(a), (b). The solid (dashed) lines indicate the second (first) order phase transitions, respectively. The dots C (T) denote the critical (tricritical) points. In figure 1(c), the dependence of the mass gaps σ and Δ on μ at $(T, \tilde{e}B) = (0, 0.3)$ GeV 2 confirms the first order

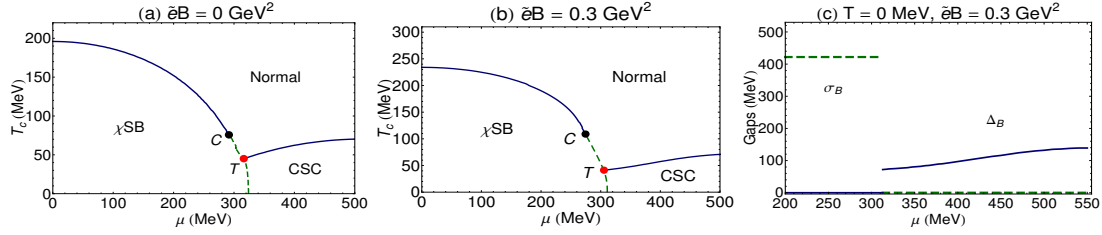


Figure 1: $T_c - \mu$ phase diagram at (a) $\tilde{e}B = 0$, (b) $\tilde{e}B = 0.3 \text{ GeV}^2$. (c) Gaps in terms of μ at $\tilde{e}B = 0.3 \text{ GeV}^2$.

transition between two phases shown in 1(b). According to the mechanism of magnetic catalysis, both T_c and μ_c grow with increasing $\tilde{e}B \gtrsim 0.45 \text{ GeV}^2$. In weak magnetic fields regime, μ_c and T_c decrease with increasing $\tilde{e}B$ in some regions of magnetic fields. This is the phenomenon of “Inverse Magnetic Catalysis” discussed in [4]. Van Alphen-de Haas oscillations are also demonstrated in this regions. In Figs. 2(c) and 2(d) the mass gaps are illustrated in terms of $\tilde{e}B$ at some arbitrary T, μ values. In 2(c) at $T = 100 \text{ MeV}$, $\mu = 300 \text{ MeV}$ a sudden jump in σ_B from the zero value to a nonzero one indicates a first order phase transition in 2(a). A similar comparison will be conducted between 2(b) and 2(d) at fixed $\mu = 460 \text{ MeV}$. For more discussions see [1].

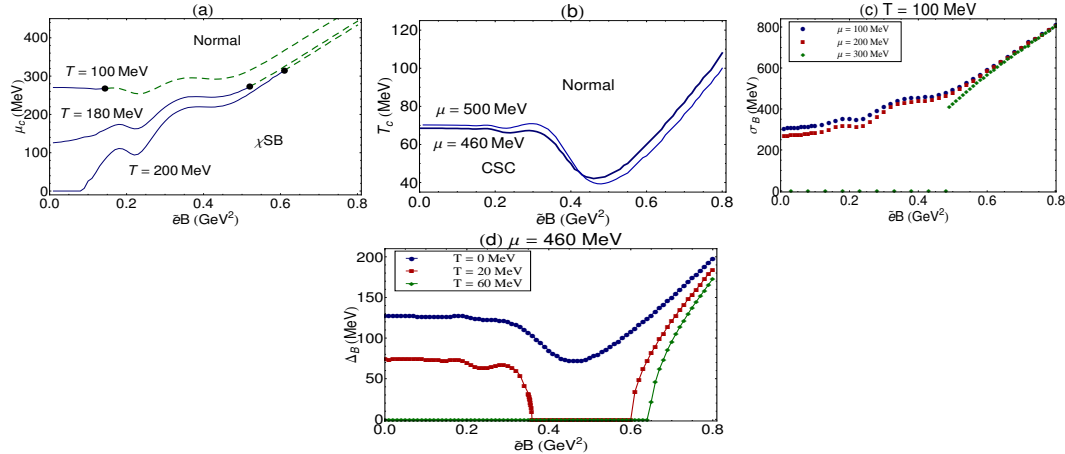


Figure 2: $\tilde{e}B$ dependence of (a) μ_c , (b) T_c , (c) σ_B and (d) Δ_B at some arbitrary constant values of T and μ .

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